UC San Diego **GUIDE FOR INSTRUCTORS**

# FROM SYMBOL MANIPULATION TO MEANING MAKING:

## A Cross-Disciplinary Video Development Project to Promote Fluency with Mathematics in Science

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Team members were Adam Burgasser, Thomas Bussey, John Eggers, Stanley Lo, Jeff Rabin, Sherry Seethaler, Laura Stevens, Haim Weizman and our late colleague, Jeff Remmel.



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## A) *Project goals on meaning making and fluency across disciplines*

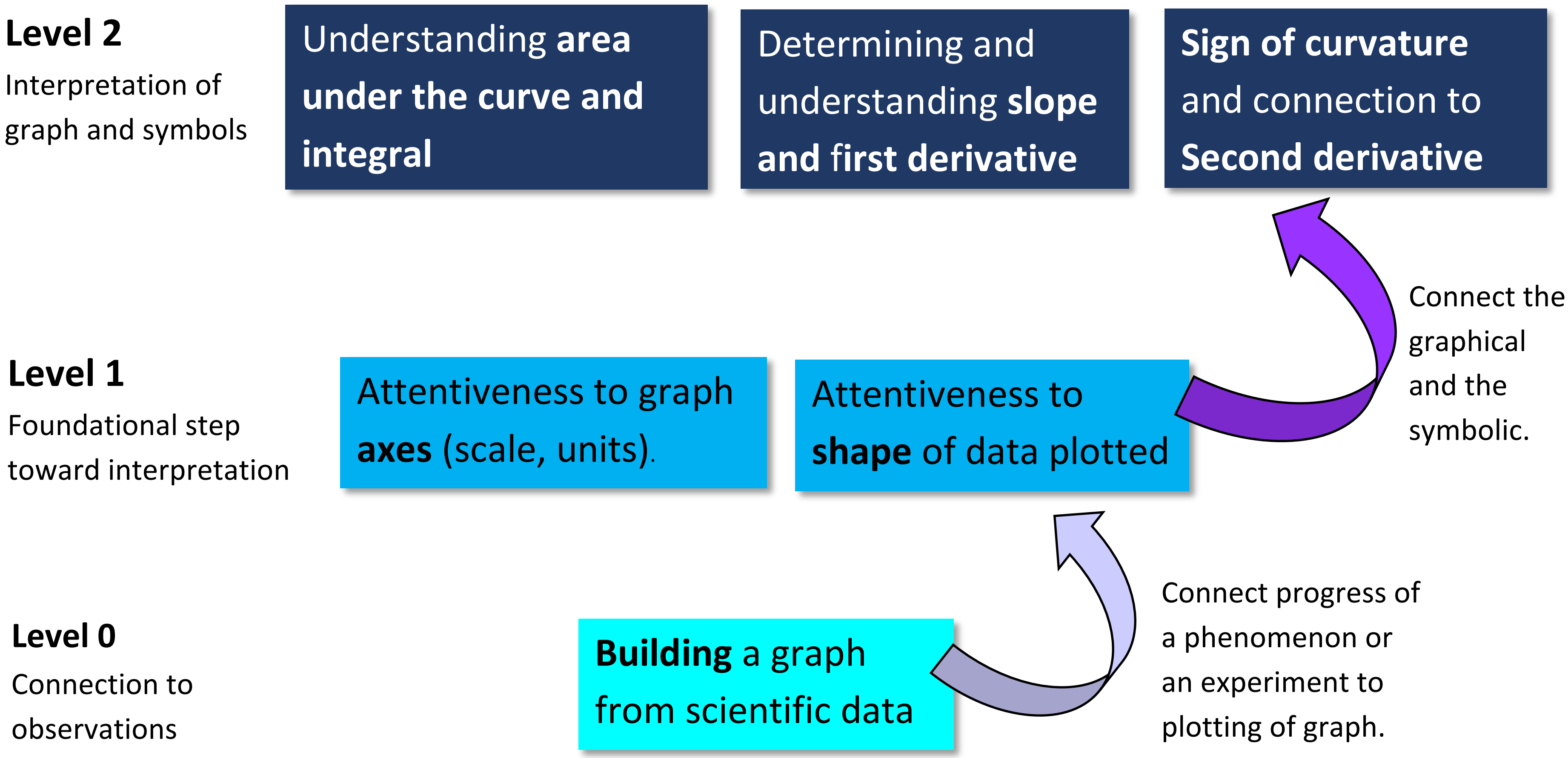
Our team had two goals for this NSF IUSE Project. The first, aligned with curriculum development, was to work across disciplines to produce a series of videos that encourage meaning making in mathematics and improve students' understanding of challenging mathematics concepts in the physics, chemistry and biology undergraduate curriculum (especially in introductory courses). During this work, our team had many conversations about the nature of meaning making and what it means to be able to think fluidly across disciplines. The table below provides examples of the shift from symbol manipulation to meaning making. As indicated by the last line of the table, this fluidity of thinking about mathematics can be related to moving up the hierarchy of Bloom's Taxonomy.

**Examples of Shifts from Symbol Manipulation to Meaning Making**

| Symbol Manipulation | Meaning Making | Skill |
|---|---|---|
| How do I do this operation? | When should I use this operation? | Application to novel problems. |
| Did I get the answer at the back of the book? | What am I solving for and does the answer seem the right order of magnitude? | Habit of mind of examining reasonableness of results. |
| What steps are we expected to show to get full credit? | Can I explain these symbols in words? What would a graph of this look like? | Moving fluently between representations. |
| **Remember** | **Understand** | **Apply** |

We also had many in-depth conversations about the many ways in which mathematical thinking is important in the sciences. One key example is the ability to develop graphical representations of experimental data and to interpret graphical representations to make appropriate inferences about the underlying phenomena. The figure on the next page provides examples of desirable learning connections between experimental data, graphical representations and mathematical abstractions.

**Examples of Levels of Understanding in Integrating Science and Mathematics**



The second goal of our project was interwoven with the first. It was to leverage the video project to deepen mathematicians' and scientists' understanding of each other's disciplines, and to lay the groundwork for using it as a tool to promote the adoption of evidence-based teaching practices.

To accomplish the goals of the project, eight colleagues representing four academic disciplines (mathematics, physics, chemistry and biology) met regularly over the course of nearly five years. Although many of these meetings were dedicated to iteratively refining the video scripts, we became a community of practice that discussed a wide range of issues in science and mathematics education, as well as scientific and mathematical thinking more generally. In addition to providing collegial support during challenging times, this work led to multiple publications and presentations. For the videos themselves, individual team members took turns leading script development and production, but each video is a collective effort.

## B) *Educational scholarship motivating the project*

*Need for an interdisciplinary approach.* The traditional undergraduate curriculum treats mathematics and science as separate entities, but these disciplines are inherently interdependent. Many have called for greater integration of the teaching of mathematics and science, more exposure to real world applications and more emphasis on ensuring that science majors develop a solid foundation in mathematics and the ability to reason quantitatively (AAAS, 2011; Bialek & Botstein, 2004; NRC, 2003, 2005a, b, 2013a; Rutherford & Ahlgren, 1991). The call for greater integration is not one-sided. The *Mathematical Sciences in 2025* report concludes, "Mathematical sciences work is becoming an increasingly integral and essential component of a growing array of areas of investigation in biology, medicine, social sciences, business, advanced design, climate, finance, advanced materials, and many more. This work involves the integration of mathematics, statistics, and computation in the broadest sense and the interplay of these areas with areas of potential application" (NRC, 2013b, p. 2). Yet, the report also concludes, **"**Many mathematical scientists remain unaware of the expanding role for their field, and this incognizance will limit the community's ability to produce broadly trained students and to attract more of them. A community-wide effort to rethink the mathematical sciences curriculum at universities is needed" (NRC, 2013b, p. 2).

*Students' challenges with mathematics in science.* Discipline-based education research has consistently shown that undergraduate students, including those who have completed their lower division mathematics requirements, can solve procedural problems, but struggle with non-routine problems and with applying mathematics in their science courses (Pepper et al., 2012; Selden, Mason & Selden, 1989; Selden, Selden & Mason, 1994; Tariq, 2008). For example, they have difficulty constructing and interpreting graphs (Potgieter, Harding & Engelbrecht, 2008). Students also often do not understand the underlying assumptions that may need to be made before applying a mathematical strategy to a science problem, and they apply mathematical strategies that are inconsistent with a particular situation (Rebello et al., 2007; Tuminaro & Redish, 2003).

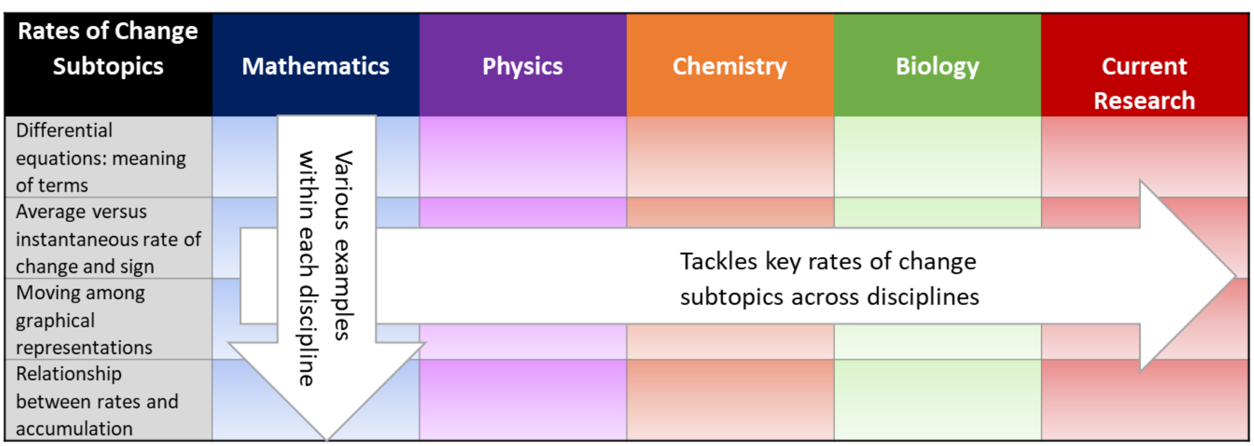
*Specific challenges with rate of change.* Students have these difficulties across mathematics topics, but one topic that causes widespread difficulty at the college level, even among strong students at selective institutions, is rate of change (Masel, 2012; Pfannkuch & Brown, 1996; Sofronas et al., 2011). Students often cannot correctly explain the meaning of terms in a differential equation (Rowland & Jovanoski, 2004). They may fail to distinguish initial rate, instantaneous rate and average rate over a time interval (Cakmakci, Leach & Donnelly, 2006). They tend to confound "amount" and "rate of change of amount" as well as constant and variable rates of change (Rowland & Jovanoski, 2004). Many also struggle with graphical aspects; for instance with kinematics graphs, failing to distinguish between distance, velocity and acceleration, and on reaction kinetics graphs, failing to distinguish between plots of rate versus time, rate versus concentration, and concentration versus time (Beichner, 1994; Bezuidenhout, 1998; Cakmakci, Leach & Donnelly, 2006). Consistent with this extensive body of research, surveys of our own students reveal that they also have many misunderstandings about rate of change after they have completed the first-year calculus sequence, such as the difference between average and instantaneous rates of change.

*Need for a focus on meaning making.* Students' errors in mathematics are often systematic and due to the difficulties of shifting from one way, or "paradigm," of thinking to another (Rowland & Jovanoski, 2004). An essential paradigm shift is from thinking of mathematics as symbol manipulation to mathematical meaning making, which includes helping students develop knowledge of when to use an operation, the ability to apply mathematics to novel problems, the habit of mind of examining the reasonableness of results, a feel for numbers (such as orders of magnitude), fluency to move between symbolic and other (such as verbal and graphical) representations, and the belief that mathematics is relevant in real world and research contexts (Schoenfeld, 1992). The transition from symbol manipulation to meaning making is difficult because of students' prior experiences in mathematics classes that emphasize procedural knowledge over conceptual understanding. Students are even willing to deny their physical experience when it conflicts with what they believe to be the right answer (Williams, 1991). Therefore efforts to promote conceptual change in students should be tied to the overarching goal of encouraging meaning making.

## C) *Rate of change videos–orientation and tips for use in STEM classes*

Our work is inspired by the discipline-based educational research findings discussed in the previous section. In summary, the traditional undergraduate curriculum treats mathematics and science as separate entities, but these disciplines are inherently interdependent. Many have called for greater integration of the teaching of mathematics and science, more exposure to real world applications and increased emphasis on ensuring that science majors develop a solid foundation in mathematics and the ability to reason quantitatively. One crosscutting concept that causes widespread difficulty among students in high school, college and undergraduate courses is rate of change. Examples of rate of change in introductory science courses are velocity and acceleration in physics, reaction kinetics in chemistry and population growth in biology. The *From Symbol Manipulation to Meaning Making* project sought to increase curricular integration of these topics by bringing together an interdisciplinary team of instructors of mathematics, physics, chemistry and biology undergraduate courses.

The team collaborated to develop videos for five themes (mathematics, physics, chemistry, biology and current research) on four rate of change subtopics (meaning of terms in differential equations, average versus instantaneous rate of change and sign, moving between graphical representations, and integration as accumulation). The resulting videos are organized into a matrix on the project website. Navigate down the columns to see all the videos on one theme (e.g. physics) or across a row to see all the videos for one subtopic (e.g. average versus instantaneous rate and sign).




The four rate of change subtopics were chosen for both their relevance in STEM and because of the specific learning challenges associated with them.

- *Differential Equations: Meaning of Terms*–Understanding differential equations entails paying close attention to the various terms and recognizing the meaning of each. Potential pitfalls include confusing parameters with initial conditions and failing to recognize the relationship between the underlying function and its first and second derivative. The videos explain initial conditions in differential equations (mathematics), and explore motion along two axes in a basketball toss (physics), the order of chemical reactions (chemistry), the basic parameters of population growth (biology), and the clearance of medications from the blood (current research).

- *Average Versus Instantaneous Rate of Change and Sign*–Understanding rate of change requires attention to both the magnitude or absolute value of the slope and the sign of the rate of change simultaneously. Initial, instantaneous and average rate of change must be distinguished symbolically, on graphs and using appropriate calculations for each. Learners may master the calculations but still struggle to explain what is happening in words. The videos explore average versus instantaneous rate and sign in a horizontal ball toss (mathematics), running on a track (physics), the chemical reactions of a hydrogen car (chemistry), yeast fermentation in bread (biology), and the separation of RNA molecules on a gel (current research).

- *Moving Among Graphical Representations*–Fluency with graphical representations entails being able to glean information from different kinds of graphs by plotting the same data in different ways, as well as being able to move between the graph of a function and the graphs of its first and second derivative. In the videos, scenarios about water in rain barrels (mathematics), a mass on a spring (physics), tie dye shirts (chemistry), populations of wolves and moose (biology), and stellar flares (current research) illustrate the usefulness of fluency with representations and help make it more intuitive.

- *Relationship Between Rate and Accumulation*–A core principle in calculus (the fundamental theorem of calculus) is that the accumulation of a quantity (determined by integration) and the rate of change in the accumulation of the quantity (determined by differentiation) are interrelated. It is common for learners to confound “amount” and “rate of change of amount,” and to fail to recognize the relationship between integration and the area under a graph. To explore these relationships, travel, volumes of solids and more (mathematics), skateboarding (physics), gas chromatography (chemistry), cell cycles and flow cytometry (biology), and the fate of pharmacological substances in the body (current research) are topics explored in the videos.

**Video Links**

**Mathematics Playlist**

Differential Equations: Meaning of Terms - This video introduces how differential equations are used to model physical processes. It distinguishes between particular and general solutions and explains the importance of the initial condition.

Average Versus Instantaneous Rate and Sign - Explore the distinction between average and instantaneous rates of change in a horizontal ball toss and understand the relationship between the observed motion and the calculations of the ball's velocity.

Moving Among Graphical Representations - The graph of a function and the graph of its derivative provide different information and it is useful to be able to move between them. This video explores the example of water flowing in and out of a rain barrel.

Relationship Between Rates and Accumulation - The integral can compute the amount of any quantity that accumulates at a known rate. This video explains why integration can be applied in such a wide range of contexts.

**Physics Playlist**

Differential Equations: Meaning of Terms - This video explores the rates describing a basketball's motion. It demonstrates how to separately analyze the distance and height and how to derive the differential equations and perform unit analysis.

Average Versus Instantaneous Rate and Sign - You ran all the way around the track and yet your average velocity is zero. How can that be? This video explores the distinction between average and instantaneous rates of change to explain that and more.

Moving Among Graphical Representations - This video explores a chaotic pendulum on a spring problem and the relationship between kinetic and potential energy. Learn how motion relates to the total energy of the system and relate the behavior to the graphical representations.

Relationship Between Rates and Accumulation - Come along for the (skateboarding) ride to learn about the work-energy theorem, displacement vectors, dot products and integration as accumulation.

**Chemistry Playlist**

Differential Equations: Meaning of Terms - How is the rate law, including the reaction order and the rate constant, for a chemical reaction determined from experimental data? Relate what is happening at the molecular level to the differential equation.

Average Versus Instantaneous Rate and Sign - This video explores core concepts in reaction kinetics, including the relationship between rate and sign, in the context of the chemical reaction that powers a hydrogen fuel cell car.

Moving Among Graphical Representations - Tie dye has a lot to do with chemical reaction kinetics because fiber-reactive dyes covalently bond to the fabric fibers. Concentration versus time and rate versus concentration graphs are useful for understanding how to optimize the process.

Relationship Between Rates and Accumulation - Gas chromatography separates molecules based on how attracted they are to the stationary phase, but why is the output a curve and what does the area under the curve represent?

**Biology Playlist**

Differential Equations: Meaning of Terms - Under certain conditions, the growth rate of a population of animals, such as seals, may be exponential. This video considers the population growth rate and the discrete and continuous equations to model the growth.

Average Versus Instantaneous Rate and Sign - The video explains how the logistic growth model and the carrying capacity are relevant concepts in understanding the role of yeast in the process of fermentation to make bread dough rise.

Moving Among Graphical Representations - Relationships between populations of predators (such as wolves) and prey (such as moose) can depend on many factors, and different graphical representations can provide unique insights into the relationships.

Relationship Between Rates and Accumulation - Flow cytometry provides information about where cells are in the cell cycle, but what exactly do those graph peaks mean, and how can you estimate the ratio of the areas under the curves when the computer is acting up?

**Current Research Playlist**

Differential Equations: Meaning of Terms - The drug clearance rate is an important clinical parameter of a drug treatment. This video discusses chemical reaction kinetics and the half-life in the context of the elimination process of a pharmaceutical from the body.

Average Versus Instantaneous Rate and Sign - How is the derivative relevant to the origin of life? This video highlights how seemingly unrelated math concepts can make it possible to track how RNA molecules evolve in a test tube.

Moving Among Graphical Representations - This video explores the graphical representations relevant to determining how much energy is released in stellar flares, which is part of the search for life on other planets.

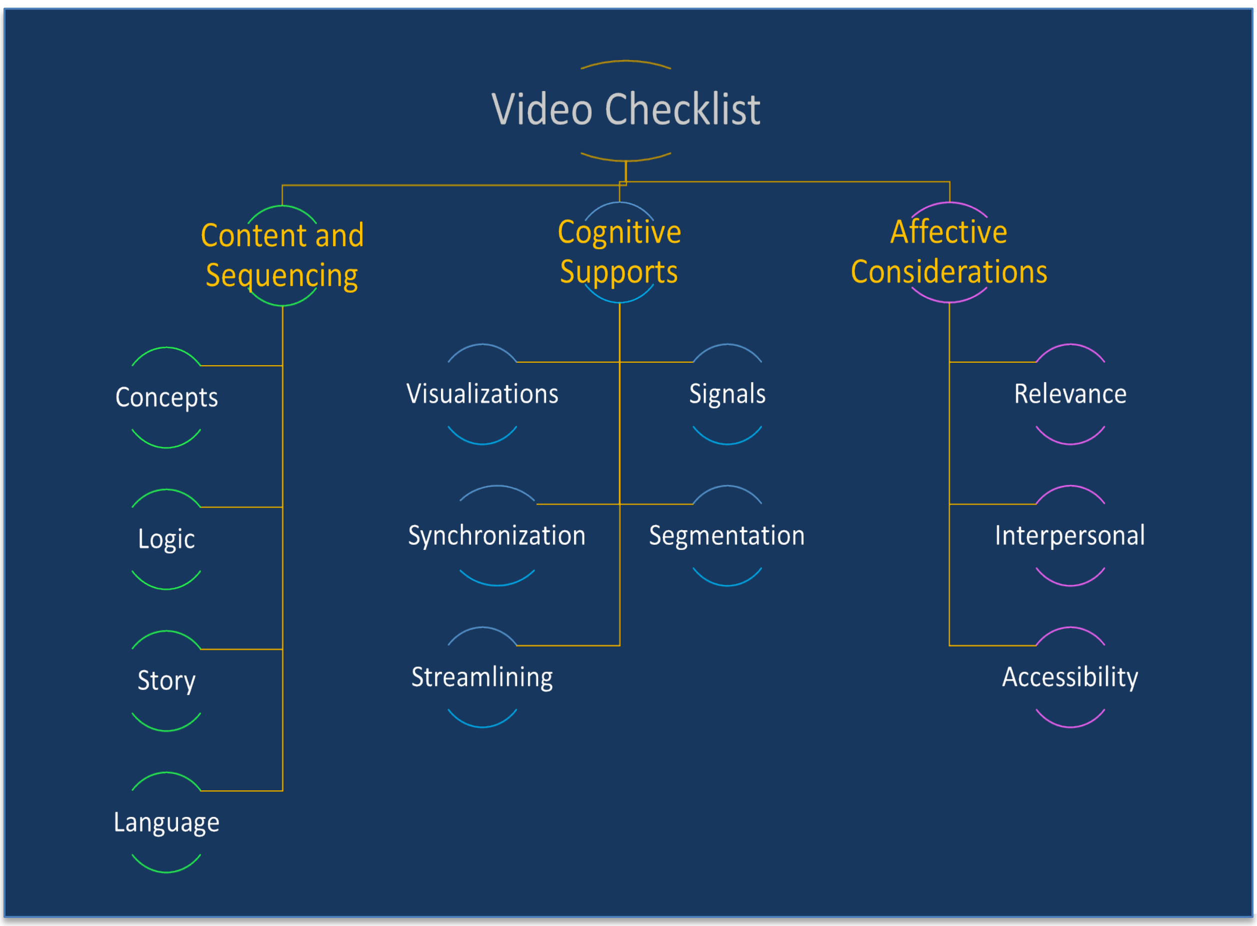
Relationship Between Rates and Accumulation - This video describes how integration makes it possible to understand how drugs are cleared from the body, and how the medication's clearance rate and plasma level may be altered during pregnancy.

The videos depict diverse, empowered learners in a wide variety of contexts that showcase the ubiquity of rate of change concepts in science and mathematics coursework, daily life and cutting-edge scientific research. They are intended as short (approximately 5-10 minutes) supplements to the curriculum to provide conceptual underpinnings that are often absent from the traditional curriculum. They can be used in class, with problem assignments or as study resources.

## D) ***Checklist to guide the design of educational videos***

As the team worked together to develop the videos, it became clear that, although a large body of education research is available to inform the design of instructional videos, it is fragmented across educational disciplinary traditions and STEM fields, and the practical lessons are not readily accessible.  Our experience with the literature and with video development inspired us to synthesize the relevant literature and translate it into recommendations for practice in the form of a user-friendly instrument.   The development of the instrument and the supporting scholarship is described in the following peer-reviewed paper.  The figure below provides an overview and the printable checklist instrument is provided on the following page.

Seethaler, S., Burgasser, A. J., Bussey, T. J., Eggers, J., Lo, S. M., Rabin, J. M., Stevens, L. & Weizman, H. (2020). A research-based checklist for development and critique of STEM instructional videos. *Journal of College Science Teaching*, *50*(1), 21-27.



## Checklist for Development and Critique of Instructional Videos

**Content and Sequencing**

- ☐ ***Concepts.*** The video clarifies the concepts it covers and makes links to students' prior knowledge, including misconceptions.
- ☐ ***Logic.*** Each successive concept in the video or video series builds on the previous ones without gaps in logic or errors.
- ☐ ***Story.*** A hook (e.g. problem or question) begins a narrative or explanatory arc that culminates in a resolution.
- ☐ ***Language.*** Tone is conversational and disciplinary terms and notation are appropriately defined and consistently used.

**Cognitive Supports**

- ☐ ***Visualizations.*** Demonstrations, animations and other visuals clarify concepts and make the invisible visible.
- ☐ ***Signals.*** Cues (e.g. arrows, highlights and verbal guidance) help students move between physical phenomena, graphs, equations, symbols and other representational forms.
- ☐ ***Synchronization.*** Graphics and narration are mutually reinforcing and well synchronized.
- ☐ ***Segmentation.*** Judicious duration, natural pauses and reiteration emphasize important points and help parse the content for the learner.
- ☐ ***Streamlining.*** Presentation avoids overburdening learners with distractions or simultaneous processing of different verbal (conflicting text and spoken) information.

**Affective Considerations**

- ☐ ***Relevance.*** Presentation tone and style are age-appropriate and motivating, and the situation or context is meaningful for the target audience.
- ☐ ***Rapport.*** Characters/audience are depicted/treated as empowered learners, and any interactions between individuals model respectful, helpful behavior.
- ☐ ***Accessibility.*** The video is of sufficient aesthetic and technical quality to meet the learning objectives and it employs Universal Design Principles for maximum accessibility.

## E) *Critique of the treatment of rate of change concepts in textbooks*

Assembling the literature review for the original NSF IUSE proposal drew attention to the shortcomings of the treatment of rate of change concepts in traditional curriculum materials. In the chemistry introductory curriculum, students encounter rate of change concepts in reaction kinetics, a topic that many students find challenging. This inspired a careful investigation of how introductory chemistry textbooks treat rate of change concepts in reaction kinetics. The treatment is highly variable across first-year texts but we identified many potential sources of confusion.

### Analyzing General Chemistry Texts' Treatment of Rates of Change Concepts in Reaction Kinetics Reveals Missing Conceptual Links

*Abstract.* Change over time is a crosscutting theme in the sciences that is pivotal to reaction kinetics—an anchoring concept in undergraduate chemistry—and students' struggles with rates of change are well documented. Informed by the education scholarship in chemistry, physics, and mathematics, a research team with members from complementary disciplinary backgrounds developed a rubric to examine how 10 general chemistry textbooks used by top producers of American Chemical Society-approved chemistry baccalaureates treat rates of change concepts in reaction kinetics. The rubric is focused around four categories of students' challenges that emerged from the literature review: (i) Fluency with graphical representations; (ii) Meaning of sign of rate of change; (iii) Distinction between average and instantaneous rates of change; and (iv) Connections between differential and integrated forms of the rate laws. The analysis reveals interesting patterns but also variability among the texts that, intriguingly, is not explained by the degree to which a text is calculus-based. An especially powerful aspect of the discipline-based education research lens is its ability to reveal missing conceptual links in the texts. For example, the analysis makes apparent specific gaps in the supports needed to help students move between representational forms (words, symbols, graphs) in the development of the differential form of the rate laws. The paper discusses the implications of the findings for chemistry instructors and chemical education research.

Seethaler, S., Czworkowski, J., & Wynn, L. (2018). Analyzing general chemistry texts' treatment of rates of change concepts in reaction kinetics reveals missing conceptual links. *Journal of Chemical Education*, *95*(1), 28-36.

## F) *Interdisciplinary conversations in a faculty learning community*

We wrote a paper to describe the discussions of our interdisciplinary group of scientists and mathematicians during our multi-year curriculum project collaboration. Our goal was to highlight subtle differences between concepts that are nominally "the same" across multiple disciplines, the confusions that both students and experts can encounter about them, and the importance of STEM instructors being aware of them. Science instructors may expect that the mathematics their students learn in prerequisite math courses will be in a ready-to-use format adapted to its applications in science, but this may not be the case. Likewise, mathematics instructors may incorrectly assume that scientific applications of calculus will use the same conceptual structures, notation, and terminology presented in calculus texts. The more STEM educators know about such disciplinary cultural differences, the more they can help their students to anticipate confusions and make connections.

**Interdisciplinary Conversations in STEM Education: Can Faculty Understand Each Other Better than Their Students Do?**

*Abstract.* Rate of change concepts from calculus are presented and applied rather differently in college mathematics, physics, biology, and chemistry classes. This is not simply a matter of pedagogical style but reflects real cultural differences between these disciplines. We describe the efforts of our interdisciplinary collaboration to understand and reconcile these differences as we designed and discussed instructional videos for students. We summarize our conversations about terminology, notation, functions, rates, units, and sign conventions across the disciplines. We present some strategies that enabled us to communicate effectively, resolve confusions and reach shared understandings. Our work has implications for others involved in collaborative interdisciplinary projects and for STEM educators.

Rabin, J. M., Burgasser, A., Bussey, T. J., Eggers, J., Lo, S. M., Seethaler, S., Stevens, L. & Weizman, H. (2021). Interdisciplinary conversations in STEM education: can faculty understand each other better than their students do?. International Journal

## G) Acknowledgments

In addition to our community of practice, Adam Burgasser, Thomas Bussey, John Eggers, Stanley Lo, Jeff Rabin, Sherry Seethaler, Laura Stevens, Haim Weizman and our late colleague, Jeff Remmel, the project benefited from a long list of contributors. This includes the major contributions of the UC San Diego Educational Technology Services team, as well as that of our student actors.

*Educational Technology Services Team*

Our three-year project stretched into five years as we struggled with pandemic-related delays in filming. We are grateful for the amazing ETS team that worked with us the entire time and so ably carried out the filming and editing and created the animations for the project, especially Seth Marshburn, Jordi Oliman and Nate Bayless. We are also grateful to Robin Martin, for overseeing the team, and to his predecessor Craig Bentley, who helped with the project formulation.

*Student Actors*

The actors (other than the faculty team members) were UC San Diego undergraduate and graduate students in STEM and STEM education. We would like to thank Leila Brasfield (chemistry video), Danica Cajigas (chemistry video), David Callahan (production assistance, chemistry video), Ernesto Calleros (math video), Kathleen Chao (current research videos), Achol Chowdhury (biology videos), Peter Emanuel (biology videos), Devin Flanagan (chemistry video), Peyton Graves (current research videos), Leah Grayson (current research video), Susan Hou (biology videos), Tina Marcroft (chemistry video), Makenna Martin (chemistry video), Dora Ogbonna (biology and mathematics videos), Riley Peacock (chemistry and current research videos), Nicole Suarez (chemistry video), Ceres Trinh (chemistry video), Ashley Warner (physics videos).

We are grateful to the many community college and undergraduate instructors who provided formative feedback and helped us disseminate the products. We would also like to thank the anonymous reviewers and editors who provided feedback on the papers we published as part of this project.

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## I) Appendix: Collection of problems for use in learning and assessment

**General/Application**

1) A machine makes widgets at a rate that depends on the ambient temperature in an open-air factory. Assuming you have an accurate hourly weather report for the next seven days and a table that lists rate of widget production by temperature.
   i) What mathematical procedure(s) could you apply to determine how many widgets your machine can produce in the next week?
   ii) Could you get a quick rough estimate? How?

**Meaning**

1) In general, what does a derivative tell you?

2) In general, what is the purpose of integration?

3) What does it mean to solve a differential equation? Why would you need to?

4) In each case, what does the slope of a graph of y versus x represent?
   i) If y is the position of a moving car and x is time;
   ii) If y is the odometer (mileage) reading of a car and x is the amount of fuel in the gas tank;
   iii) If x is the price of a widget and y is the number that a store can sell at that price;
   iv) If y is the volume of a balloon and x is its radius;
   v) If y is the net growth rate of a rabbit population and x is the number of wolves in their environment.

5) In each case, what does the area under the curve of the graph of y versus x represent?
   i) If y is the velocity of a moving car and x is time;
   ii) If y is the force acting on an object and x is its position along a number line;
   iii) If x is on an IQ scale from 0 to 200 and y at location x is the number of people with IQ scores within a small distance $\Delta x$ of x, divided by $\Delta x$;
   iv) If y is the density of cars (cars per mile) at a location x miles along a highway.

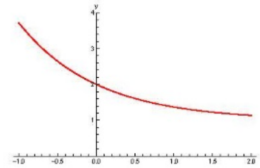
**Related to Specific Videos**

**<u>Mathematics</u>**

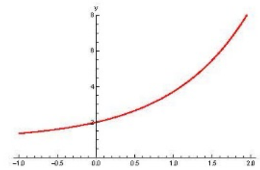
(*Differentiation*)

(4 points) 1. Which of the following could be the graph of a solution of the differential equation $\frac{dy}{dt} = \frac{y}{2}$?

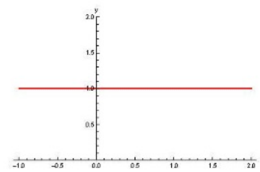
(a) GRAPH A



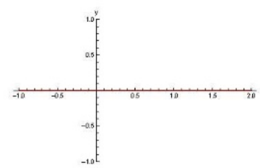
(b) GRAPH B



(c) GRAPH C



(d) GRAPH D



(*Integration*)

1. The density of bacteria at a point on a petri dish depends only on the distance r of that point from the center. Suppose this density (cells per square centimeter) is given by a function f(r), with r in centimeters.

   i) How would you compute the total number of bacteria within a circle of radius R about the center?

   ii) Some other students have suggested the following formulas for this number. Do you agree with any of them, and why?

      a) $\pi R^2 f(R)$

      b) $\int_0^R f(r)dr$

      c) $\int_0^R \pi r^2 f(r)dr$

      d) $\int_0^R 2\pi r f(r)dr$

      e) $\int_{-R}^R 2\pi R f(r)dr$

      f) Something else?

**Physics**

*(Signs)*

1. A car is driving along the x axis when the driver suddenly applies the brakes. What additional information do you need to determine whether the car's acceleration is positive or negative? More than one answer may be correct.
   a) Whether the car's x coordinate is positive or negative
   b) Whether the car's velocity is positive or negative
   c) Whether the car's x coordinate is increasing or decreasing
   d) None: the acceleration must be negative
   e) Whether the force acting on the car is to the left or the right

Please explain your choice.

*(Integration--Skateboarding)*

1) In which case would you expect an increase of speed? (Where v is velocity and F is force.)
   a) v parallel to F
   b) v antiparallel to F
   c) v perpendicular to F
   d) doesn't depend on direction

2) In which case would you expect an increase of kinetic energy? (Where v is velocity and F is force.)
   a) v parallel to F
   b) v antiparallel to F
   c) v perpendicular to F
   d) kinetic energy is conserved, so it can't increase or decrease

3) In which case would you expect a decrease of kinetic energy? (Where v is velocity and F is force.)
   a) v parallel to F
   b) v antiparallel to F
   c) v perpendicular to F
   d) kinetic energy is conserved, so it can't increase or decrease

4) In which case is C = A dot B positive (where A and B are vectors)?
   a) A parallel to B
   b) A antiparallel to B
   c) A perpendicular to B
   d) it depends on the magnitude of A and B

5) In which case is C = A dot B equal to zero (where A and B are vectors)?
   a) A parallel to B
   b) A antiparallel to B
   c) A perpendicular to B
   d) it depends on the magnitude of A and B

6) When the skateboarder was moving in a circle, in which direction was his friend applying a force on him?
   a) directed toward center of circle
   b) directed away from center of circle
   c) directed tangent to circle in same direction of v
   d) directed tangent to circle in opposite direction of v

7) Why was there no work done on the skateboarder when he was moving in a circle?
   a) his speed didn't change
   b) his velocity didn't change
   c) his direction of motion didn't change
   d) there was no force applied to him

8) If we want to increase the kinetic energy of the person on the skateboard, how should we push it?
   a) can only push exactly in direction of motion
   b) can only push exactly perpendicular
   c) can only push exactly opposite direction
   d) can push in wide range of directions, as long as there is a component in direction of motion

9) You have force F that depends in a complicated way on position x. How would you compute the work done on an object by that force over a displacement **D**?
   a) $W = \mathbf{F}(\mathbf{D})\cdot\mathbf{D}$
   b) $W = (\mathbf{F}(\mathbf{D})-\mathbf{F}(0))\cdot\mathbf{D}$
   c) $W = \int_0^D \mathbf{F}(\mathrm{x}) \cdot \mathrm{d}\mathbf{x}$
   d) $W = (\mathrm{d}\mathbf{F}(x)/\mathrm{d}x)\cdot\mathbf{D}$
   e) $W = \mathbf{F}(\mathbf{D})\cdot\int \mathrm{d}\mathbf{x}$
   f) $W = \int_0^D \mathrm{d}\mathbf{F}(\mathrm{x}) \cdot \mathrm{d}\mathbf{x}$
   g) $W = \int_0^D \mathrm{d}\mathbf{F}(\mathrm{x}) \cdot \mathbf{D}$

10) Two students are trying to figure out how to compute the change of kinetic energy of an object that has been acted on by a force that depends on position. The notation <F> means average force.

    *Student A:* Change in kinetic energy is work, and work is force dot displacement, so I would just use the average force and then take the dot product with the displacement, like this: $W=<\mathbf{F}>\cdot\mathbf{D}$

*Student B:* Hmm, I'm not sure that's right, doesn't the problem say force changes with position? I think we need to compute a little bit of work along each step, then add all that work up. Isn't that an integral?
*Student C:* Yeah, it's an integral. I think we integrate the magnitude of the average force over position, like this: W = $\int_0^D < \mathrm{F} > \mathrm{dx}$
*Student A:* What happened to the dot product?
*Student B:* Yeah, something isn't right here. A little work looks like dW = **F**(x)·d**x**, so I think the integral might be W = $\int_0^D \mathbf{F}(\mathrm{x}) \cdot \mathrm{d}\mathbf{x}$
*Student A:* Doesn't that end up being the same thing wrote I did the first time?

Which students are thinking correctly about this, and which students are not? Why? Are there special cases in which one or more students might be actually be right?

**Chemistry**

*(Tie dye--Graphing)*

1) If the graph of reactant concentration versus time for a chemical reaction is linearly decreasing, what does that say about the rate versus concentration?

a) The rate is increasing linearly.
b) The rate is increasing non-linearly.
c) The rate is constant.
d) The rate is decreasing linearly.
e) The rate is decreasing non-linearly.

*(Bromine reaction--Differential equations)*

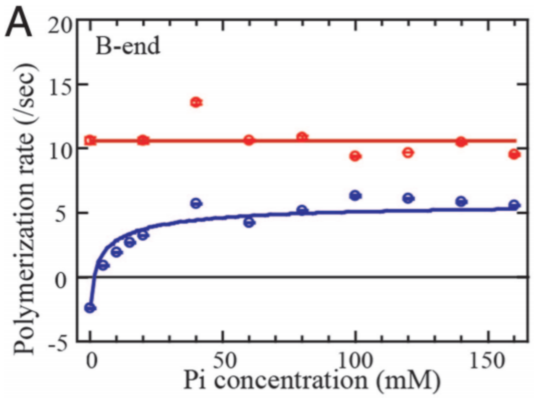
2) Which of the following about the rate constant for a chemical reaction is false.

a) It is the slope in the differential form of the rate law.
b) It is dependent on the reactant concentration.
c) It is characteristic of a particular chemical reaction.
d) It is dependent on the presence of a catalyst.
e) It is dependent on temperature.

**Biology**

(*Average and Instantaneous rates of change--Yeasts*)

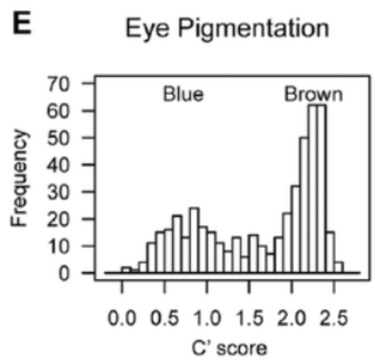
1. This question refers to Figure 2a in the paper:
https://pollardlab.yale.edu/sites/default/files/files/bibliography/223.pdf




i) Which line has the higher average rate of change? Red or blue?
iii) For the blue line, which point has the highest instantaneous rate of change? 5, 10, 50, or 100 mM?

(*Flow Cytometry--Integration*)

2. This question refers to Figure 1e in the paper:
https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0048294




i) What is the meaning of "area under the curve" here?